Sentiment Effect and Market Portfolio Inefficiency

K. Victor Chow
West Virginia University
Rudolf F. Klein
Sweet Briar College

Abstract

We apply Marginal Conditional Stochastic Dominance (MCSD) tests to returns on sentiment-beta sorted portfolios and sentiment-arbitrage portfolios, constructed using the Baker and Wurgler (2007) index of sentiment levels. The theory of MCSD demonstrates that, if one (mutually exclusive) subset of a core portfolio dominates another, conditional on the return distribution of the core portfolio, then the core portfolio is inefficient for all utility-maximizing risk-averse investors. Based on returns on the U.S. equity market, we show that both positively and negatively sentiment sensitive stocks are conditionally and stochastically dominated by sentiment insensitive stocks. Moreover, we find dominance among sentiment-arbitrage portfolios, constructed with positively sensitive vs. insensitive, insensitive vs. negatively sensitive, and positively vs. negatively sensitive stocks. Therefore, we conclude that the market portfolio is stochastically inefficient.

Keywords: Investor Sentiment, Market Portfolio Efficiency, Stochastic Dominance.
JEL Classification: D03, G11, G14

Introduction

Testing market portfolio efficiency relative to different sets of portfolios has been a main theme of financial research. We propose an approach that avoids the major drawbacks of other methods and manages to give clear answers, in a simple and less restrictive way. Using Marginal Conditional Stochastic Dominance (MCSD) tests, we are able to prove the existence and importance of the *sentiment effect* and associate it to a case of inefficiency of the market portfolio.

Mean-variance efficiency tests are the pioneers in this area.[1] For instance, Gibbons, Ross, and Shanken (1989) develop a multivariate *F*-test that checks whether the intercepts are jointly equal to zero. This test is easy to implement and offers a nice economic interpretation (in terms of *Sharpe ratios*), but its theoretical validity depends on the normality assumption of the disturbances. MacKinlay and Richardson (1991), Zhou (1993), and Richardson and Smith (1993) show that this assumption does not hold empirically.

The theory of stochastic dominance, developed initially by Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970), and reassessed by Levy and Sarnat (1984) and Levy (1992), was largely augmented by empirical tests for stochastic dominance efficiency, as developed by Post (2003) or Kuosmanen (2004).

Best et al. (2000) show that the U.S. *value* portfolios second-order stochastically dominate (SSD) the U.S. *growth* portfolios (for the interval July 1978 – June 1998) and conclude that this result is inconsistent with market portfolio efficiency. Nevertheless, Post and Vliet (2004) underline the sensitivity of the SSD results to sampling variation (as the SSD rule considers the whole sample distribution), and reject the aforementioned conclusion. They point out that the market portfolio is actually inefficient when extending the period to July 1968 – June 1998.

Chou and Zhou (2006) use a *bootstrap* method to test the mean-variance efficiency of a given portfolio, and claim that the method provides more reliable and robust results, but in a computationally-expensive manner.

Post and Versijp (2007) apply multivariate statistical tests for stochastic dominance efficiency of a given portfolio and obtain that the market portfolio (proxied by the CRSP all-share index) is significantly mean-variance inefficient relative to ten market beta-sorted portfolios. A strategy of buying low beta stocks, while selling high beta stocks can lead to a higher Sharpe ratio compared to that of the market (i.e. low beta stocks are underpriced and high beta stocks are overpriced in the mean-variance framework). They blame this inefficiency on the tail risk, not captured by variance. The mean-variance beta underestimates the tail risk for low beta stocks and overestimates the tail risk for high beta stocks.

An earlier article by Post and Vliet (2006) concludes that the same proxy for the market portfolio (i.e. the value-weighted CRSP index), is also mean-variance inefficient relative to benchmark portfolios formed on *size*, *value* and *momentum*, for the same time period: January 1933, to December 2002.

We apply the MCSD tests in the context of a multifactor linear model, so we need to turn our attention to this type of models. Multifactor models, alternatives to the traditional Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM), have become popular in recent decades. For instance, an extension of the CAPM to a multi-country case is the widely known International CAPM.[2] Other extensions, to a multi-period economy, are the Intertemporal CAPM and Consumption-based CAPM.[3]

A widely cited multifactor model is the Fama and French Three Factor Model (1992, 1993, 1996 and 1998). The model is considered a special case of the Arbitrage Pricing Theory, as developed by Ross (1976). It considers the existence of three factors that determine the asset pricing, but those factors are only mimicked by three well diversified portfolios: market, SMB (i.e. Small minus Big, market capitalization) and HML (i.e. High minus Low, book-to-market ratio). Fama and French (1992, 1993, 1996 and 1998) also claim that a series of the so-called *anomalies* can be explained using their model.[4] They see higher returns (i.e. excess returns between dominated and dominating assets) as compensation for taking on more risk (i.e. systematic risk factors that are therefore priced).

The Carhart (1997) model extends the Fama-French model, by including a fourth factor: *momentum*. The *momentum* effect of Jegadeesh and Titman (1993, 2001), Chan, Jegadeesh, and Lakonishok (1996), Rouwenhorst (1998), and others, indicates that average returns on the prior best performing stocks (the so-called *winners*) exceed those of the prior worst performing stocks (the so-called *losers*), and thus short-term past returns have predictive power over future returns.

[1] *For an excellent review of the early literature on mean-variance efficiency tests, see Shanken (1996).*
[2] *See for example Solnik (1974), Stulz (1981), Adler and Dumas (1983), Black (1990), and DeSantis and Gérard (1998).*
[3] *See Merton (1973) and Breeden (1979), respectively.*
[4] *However, a number of studies blame biases in the empirical methodology for the documented anomalies. Lo and MacKinlay (1990), MacKinlay (1995), Knez and Ready (1997), and Loughran (1997) argue that the empirical evidence can actually result from data-snooping biases such that the anomalies are sample dependent. Therefore, they are unlikely to be observed out-of-sample.*

Recently, a number of articles have been inspecting the effect of investor sentiment on common stock returns.[5] Baker and Wurgler (2006, 2007) examine investor sentiment as another determinant of stock returns. They construct sentiment indexes (hereafter denoted as BW) and find that returns are affected by the level of pessimism/ optimism, even when controlled for the Fama and French factors.[6]

Glushkov (2006) tests whether exposure to sentiment is a priced factor, namely whether investors demand premium for holding stocks with more exposure to sentiment. He develops a sentiment factor, taking the first principal component of different measures of investor sentiment (similar to BW). He finds a sentiment beta after controlling for risk factors associated with *market*, *size*, *value* and *liquidity*. The relationship between sentiment betas and returns turns out to be inverse U-shaped, which means that *low* and *high* beta stocks tend to underperform the *near-zero* beta stocks. The under-performance of extreme beta portfolios (with no significant difference between them), compared to near-zero sentiment beta portfolios, is also manifested for sub-periods, which means, he concludes, that there is no reason to think about sentiment as a risk factor.

This paper applies MCSD tests in order to examine the existence of a *sentiment* effect and to inspect the efficiency of the market portfolio. Unlike the traditional SSD rules of comparing unconditional return distributions of assets independently, MCSD considers the joint nature between assets and the market.[7] The MCSD theory, originally developed by Shalit and Yitzhaki (1994), focuses on necessary and sufficient conditions to improve investors' expected utility of wealth, by marginally reallocating the assets in their portfolios (i.e. by increasing the share of the dominating assets on the account of the dominated ones).[8] Specifically, suppose that the market portfolio can be decomposed into a set of mutually exclusive sub-portfolios according to the stocks' sensitivity to investor sentiment: positively sensitive (β_+^s), insensitive (β_0^s), and negatively sensitive (β_-^s) stocks. Conditional on the return distribution of the given market portfolio, if for instance β_0^s marginally and stochastically dominates β_+^s according to the MCSD criteria, then we conclude that the market portfolio is stochastically inefficient, in that risk-averse investors prefer to hold a re-allocated portfolio by selling β_+^s stocks and purchasing more of β_0^s stocks.

To examine the *sentiment* effect, we sort all the NYSE, AMEX, and NASDAQ stocks (that do not have missing values during the regression period), according to their sentiment betas, (based on the BW sentiment levels index), after controlling for *market*, *size*, *value*, and *momentum* factors. Thus, we form three sentiment-beta sorted portfolios. We also construct three sentiment-arbitrage portfolios with positively sensitive vs. insensitive, insensitive vs. negatively sensitive, and positively vs. negatively sensitive stocks, dependent on different levels of investor sentiment. Employing a statistical inference MCSD test developed by Chow (2001), we find that both positively and negatively sentiment sensitive stocks are conditionally and stochastically dominated by sentiment insensitive stocks. Moreover, we find dominance among the sentiment-arbitrage portfolios, which proves once again that the market portfolio is inefficient relative to portfolios formed on investor sentiment.

The paper is organized as follows. Section II reviews the MCSD ranking rule and its statistical inference procedures. In section III, we describe the data, the empirical hypothesis and we present our main results. Section IV draws brief conclusions.

Marginal Conditional Stochastic Dominance Test

Traditional portfolio selection models such as stochastic dominance, mean-variance, and performance measures, rank portfolios unconditionally and independently. These approaches are appropriate for individual asset selection, but they are unable to effectively solve the problem of improving portfolio holding by changing asset allocation in the portfolio. Shalit and Yitzhaki (1994) argue that, in reality, investors usually optimize their portfolios by marginally changing asset allocation, without altering the core portfolio.

Let a diversified core-portfolio, such as a market index portfolio, be decomposed into a set of *n* mutually exclusive sub-portfolios according to a fundamental metric (sentiment sensitivity, in our case). The return on the core-portfolio can be written as $r_m = \sum_{s=1}^{n} w_s r_s$, where r_s is the return of the *s*-th sub-portfolio, and $\sum_{s=1}^{n} w_s = 1$. As mentioned above, Shalit and Yitzhaki (1994) claim that when investors maximize their expected utility, they normally reallocate securities marginally, without altering their core holdings. So, what is the condition such that investors are willing to marginally change their asset allocation to optimize their utility?

Assume that investors are non-satiated and risk-averse (i.e. their preference functions are such that: $U'>0$ and $U''<0$). Also, they are maximizing their expected utility, *E(U(W))*, where $W = 1 + \sum_{s=1}^{n} w_s r_s$ is the final wealth (assuming an initial wealth of $1).

Then, a sub-portfolio *p* dominates another sub-portfolio *q*, given the core-portfolio, if the following inequality holds for all investors:

$$\frac{dE(U(W))}{dw_p} = EU'(W)(r_p - r_q) \geq 0 \qquad (1)$$

Shalit and Yitzhaki (1994) formulate the necessary and sufficient conditions such that inequality (1) holds, in terms of *Absolute Concentration Curves* (ACCs), which are defined as the cumulative expected returns on assets / sub-portfolios, conditional on the return on the core-portfolio. Since the concept of ACC is less familiar in the financial literature, Chow (2001) reformulates these conditions in a relatively simple way, as follows:

Theorem 1. For all risk-averse investors, the inequality (1) holds if and only if

[5] *See for example Lee, Shleifer and Thaler (1991), Mitchell, Pulvino and Stafford (2002), Qiu and Welch (2004) and Brown and Cliff (2004, 2005).*
[6] *Liu (2006) studies the effect of sentiment on stock market liquidity. She finds that BW is a significant factor for liquidity, but only for some assets (for instance, portfolios with high sentiment beta), not for the market as a whole.*
[7] *For a comprehensive review of the advantages of the SSD versus conventional mean-variance analysis, see for instance Chow et al. (2008).*
[8] *Jewitt (1987) derives conditions that all risk-averse individuals prefer one particular sub-portfolio over another, given that they hold the rest of the portfolio.*

$$\int_{-\infty}^{\tau_\rho^m}\int_{-\infty}^{\infty} r_p\, f(r_p, r_m) dr_m dr_p \geq \int_{-\infty}^{\tau_\rho^m}\int_{-\infty}^{\infty} r_q\, f(r_q, r_m) dr_m dr_q \quad (2.1)$$

or,

$$E\left(r_p - r_q \middle| r_m \leq \tau_\rho^m\right) \geq 0, \quad (2.2)$$

for all ρ, where $0 \leq \rho \leq 1$; E is the expectation operator; $\tau_\rho^m = F_m^{-1}(\rho)$; $F_m^{-1}(\rho)$ is the inverse cumulative density function of r_m, corresponding to abscissa ρ.

From equation (1) and Theorem 1, it is clear that the existence of MCSD for any pair of sub-portfolios ensures that the core-portfolio is not optimal, because risk-averse investors are able to increase their expected utility through a reallocation between the pair of sub-portfolios. The application of Theorem 1 to the test of market portfolio stochastic efficiency is straightforward. Let the market portfolio be the core-portfolio, while the sub-portfolios are constructed as the sorted mutually exclusive groups of assets according to information about the *sentiment* metric. Our null hypothesis is that there is no MCSD (implying that the core-portfolio is efficient). Applying MCSD tests to pairwise comparisons of sub-portfolios, if there is at least one MCSD, we then reject the null hypothesis and conclude that the market portfolio is inefficient. Notably, when $\rho = 1$, the inequalities (2) are equivalent to the difference between the mean returns on sub-portfolios p and q.

To test for MCSD, a transformation of inequality (2.1) is necessary. Without loss of generality, let $I_m^{\tau_\rho^m}$ be an indicator variable such that $I_m^{\tau_\rho^m} = 1$, if $r_m \leq \tau_\rho^m$, and $I_m^{\tau_\rho^m} = 0$, otherwise. Then, the inequality (2.2) can be written as:

$$E(r_p I_m^{\tau_\rho^m}) \geq E(r_q I_m^{\tau_\rho^m}) \quad (3)$$

To apply the statistical inference procedure of MCSD, we begin by selecting a set of target returns, $\{\tau_t^m = \hat{F}_m^{-1}(\rho_t) \mid t = 1,\ldots k\}$, corresponding to the abscissas $\{\rho_t \mid t = 1,\ldots,k\}$. For instance, in the case of deciles, $k = 10$ and $\rho_1 = 0.1$, $\rho_2 = 0.2$, …, $\rho_{10} = 1.0$. Further, let

$$\Phi_{p-q}^{\tau_t^m} = E(r_p I_m^{\tau_t^m}) - E(r_q I_m^{\tau_t^m}) \quad (4)$$

There are three possible outcomes from the MCSD test: equality ($\Phi_{p-q}^{\tau_t^m} = 0$ for all t) ; dominance ($\Phi_{p-q}^{\tau_t^m} > 0$ for some t, but $\Phi_{p-q}^{\tau_t^m} = 0$ for the rest of t, or $\Phi_{p-q}^{\tau_t^m} < 0$ for some t, but $\Phi_{p-q}^{\tau_t^m} = 0$ for the rest of t); and non-comparability ($\Phi_{p-q}^{\tau_t^m} > 0$ for at least one t and $\Phi_{p-q}^{\tau_t^m} < 0$ for at least one t). Since conventional goodness-of-fit measures (e.g. *Chi-square* and *F*-test) are unable to distinguish between dominance and non-comparability when the null hypothesis of equality is rejected, a multiple comparison test becomes necessary. It is also important to note that, using empirical quantiles from the market return sample as targets may involve sampling variation from the population quantiles. However, data snooping bias is limited (Chow, 2001).

By employing the target approach, the statistical inference of MCSD is simple and straightforward. Given a set of N random sample returns, $\{(r_{p1}, r_{q1}, r_{m1}), \ldots, (r_{pN}, r_{qN}, r_{mN})\}$, the sample estimates of MCSD ordinates can be expressed as:

$$\hat{\Phi}_{p-q}^{\tau_t^m} = N^{-1}\sum_{i=1}^{N} (r_{pi} I_{mi}^{\tau_t^m}) - (r_{qi} I_{mi}^{\tau_t^m}) \quad (5)$$

Chow (2001) shows that the sampling distribution of $\sqrt{N}(\hat{\Phi}_{p-q}^{\tau_t^m} - \Phi_{p-q}^{\tau_t^m})$ is normal, and further provides a full variance-covariance structure of estimates.[9] Importantly, one may easily perform a statistical inference for MCSD by testing a set of *Z*-statistics under the null hypothesis $H_0 : \{\Phi_{p-q}^{\tau_t^m} = 0 \mid t = 1,\ldots,k)$. The test statistic can be written as:

$$Z_{p-q}^{\tau_t^m} = \sqrt{N}\frac{\hat{\Phi}_{p-q}^{\tau_t^m}}{S_{p-q}^{\tau_t^m}}, \quad (6)$$

for $t=1,\ldots,k$, where $S_{p-q}^{\tau_t^m}$ is the sample standard deviation. To control for the size of the above multiple comparison procedure, it is necessary to adjust the critical value of the test. Using the Studentized Maximum Modulus (*SMM*) approach, the asymptotic joint confidence interval of at least 100(1 - α) percent for a set of MCSD estimates is:

$$Z_{p-q}^{\tau_t^m} \pm SMM(\alpha;k;\infty) \quad \text{for } t = 1,2,\ldots,k, \quad (7)$$

where $SMM(\alpha;k;\infty)$ is the asymptotic critical value of the α point of the *SMM* distribution with parameter k and ∞ degrees of freedom. Thus, the empirical MCSD rules using the above inference procedure are summarized as follows:

(a) An asset/ sub-portfolio p dominates an asset/ sub-portfolio q, if at least one strong inequality holds, $Z_{p-q}^{\tau_t^m} > +SMM(\alpha;k;\infty)$, and no $Z_{p-q}^{\tau_t^m}$ statistic has a value less than $-SMM(\alpha;k;\infty)$.

(b) An asset/ sub-portfolio p is dominated by an asset/ sub-portfolio q, if at least one strong inequality holds, $Z_{p-q}^{\tau_t^m}$ <

[9] *It is assumed that sample returns of each portfolio are identically and independently distributed. To generate i.i.d. sample returns, one may randomize the return data.*

$-SMM(\alpha;k;\infty)$, and no $Z_{p-q}^{\tau_t^m}$ statistic has a value greater than $+SMM(\alpha;k;\infty)$.

(c) No dominance exists otherwise.[10]

Chow (2001) shows that although the MCSD test is conservative in nature, it has power to detect dominance for samples with more than 300 observations, and is robust under both homoskedasticity and heteroskedasticity assumptions.

Empirical Results

We consider the following setting for a multifactor linear model:

$$r_t^i - r_t^f = \alpha^i + \gamma^i RM_t + s^i SMB_t + h^i HML_t + m^i Mom_t + \beta^i Sent_t + \varepsilon_t\,, \qquad (8)$$

where $r^i - r^f$ represent the excess returns on all common stocks listed on the NYSE, AMEX, and NASDAQ (that do not have missing values during the regression period), over the one-month Treasury bill rate (from Ibbotson Associates); *RM*, *SMB* and *HML* are the Fama and French factors: *RM* is the market risk premium, *SMB* is Small Minus Big (size), and *HML* is High Minus Low (book-to-market), while *Mom* is the momentum factor; *Sent* is the Baker and Wurgler (2007) sentiment levels index.

Monthly data, ranging from January 1966 to December 2010, are obtained from the following sources: stock returns – from the CRSP database, *market risk premium*, *size*, *value* and *momentum* factors – from Kenneth French's data library (at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/), and the index of sentiment levels – from Jeffrey Wurgler's website (http://pages.stern.nyu.edu/~jwurgler/).

As shown in Table 1, the five regressors in equation (8) are not highly correlated. Their correlation coefficients range, in absolute value, from 0.01 to 0.31. Note that the sentiment level is standardized for the July 1965 – December 2010 interval (see Baker and Wurgler, 2007), resulting in close to zero (0.018) mean and close to one (0.993) standard deviation for our slightly different time period.

To form the sentiment-beta portfolios, we run regression (8) for the first twenty-four months and then we sort all assets (that do not have missing values during that particular period) on sentiment betas. The top 25, middle 50 and bottom 25 percent are considered the positively sensitive (β_+^s), insensitive (β_0^s), and negatively sensitive (β_-^s) stocks, respectively. Next, we repeat the procedure using a rolling-window approach (the overlapping interval is twelve months).

Table 2 reports the descriptive statistics for these three portfolios. Importantly, as shown in the lower half of the table, the pairwise differences between the mean returns on the portfolios are not significantly different from zero. So, we cannot perform any first-order sorting. For each of them, the monthly excess returns are, on average, slightly above 0.7 percent.

At the second order, the MCSD tests find significant dominance (see Table 3): both positively and negatively sentiment sensitive stocks are conditionally and stochastically dominated by sentiment insensitive stocks. This inverse U-shaped pattern is in line with Glushkov (2006). The dominance is significant for the first seven targets.[11] For instance, for the first five targets, conditional on the return on the market portfolio, β_0^s outperforms β_+^s by roughly 0.4 percent per month. That is to say, on the downside of the market, the insensitive stocks surpass the positively-sensitive stocks (the same goes for the negatively-sensitive stocks). These results prove the existence of a *sentiment* effect. More notably, the market portfolio is found inefficient relative to the sentiment-beta portfolios.[12] Thus, investors are able to improve their expected utility by marginally changing the weights on their portfolios (i.e. by increasing the share of the insensitive versus the sentiment-sensitive stocks). The dominance fades out towards the upper deciles of the market distribution, but does not reverse significantly, which does not affect the importance of our results.

If we decompose the time series into three sub-periods: *high*, *medium* and *low* (i.e. corresponding to values greater than/ between/ less than the 3rd/ 1st quartile of the BW levels index), we notice that highly significant excess returns on the sentiment-beta portfolios are obtained, on average, only during the *medium* sentiment periods. Less significant results correspond to *low*, while for *high*, on average, there are no significant excess returns (see Table 4). For example, the insensitive stocks exhibit mean excess returns of about 0.8 percent per month for *medium* and 1 percent for *low* (statistically significant at 1% and 10%, respectively), while for *high*, they are statistically insignificant. Intuitively, during *high* sentiment periods, stocks are overpriced, causing lower subsequent returns (and vice versa for *medium*). Apparently, stocks rebound slower after *low* sentiment periods, compared to the more sudden rise that follows immediately after *medium* periods.

What if we condition on the level of sentiment and ignore the *medium* periods: can we still find significant stochastic dominance between sub-portfolios? The answer is affirmative. Suppose that we check the level of sentiment (in the month that follows the sorting on sentiment-betas), and decide to pursue the following three simple investment strategies for the next twelve months: if the sentiment level is *high* (i.e. greater than the third quartile of the BW levels index), then long β_+^s and short β_0^s, long β_-^s and short β_0^s, and finally, long β_-^s and short β_+^s. If the sentiment level is *low*, the strategies reverse signs. Otherwise, if the sentiment level is *neutral*, we take no action. Next, we repeat the steps using overlapping intervals, on twelve months.

Thus, we construct three sentiment-arbitrage portfolios: β_+^s *vs.* β_0^s, β_0^s *vs.* β_-^s and β_+^s *vs.* β_-^s.[13] Table 5 reports the summary statistics. Not surprisingly, we see a primary ranking that favors the two portfolios that contain the insensitive stocks, over the portfolio β_+^s *vs.* β_-^s. Mainly, β_0^s *vs.* β_-^s outperforms β_+^s *vs.* β_-^s by 0.165 percent monthly, significant at the 1% level.

The MCSD test results (reported in Table 6) confirm the substantial dominance of the portfolio β_0^s *vs.* β_-^s over β_+^s *vs.* β_-^s, for all ten targets, and by approximately 0.13 percent per month, on average. Less significantly, at the 10% level, portfolio β_+^s *vs.* β_0^s dominates β_+^s *vs.* β_-^s, while between β_+^s *vs.* β_0^s and β_0^s *vs.* β_-^s there is hardly any preference. However, further test results (not reported in this study) indicate that if we exclude the time period around the late-2000s "Great" Recession, the patterns of dominance of β_0^s *vs.* β_-^s and β_+^s *vs.* β_0^s over β_+^s *vs.* β_-^s are more balanced.

[10] *There are two possible cases: (1) all statistics are neither greater than +SMM, nor less than –SMM. In this case, we fail to reject the null hypothesis that the two distributions are equal, and (2) if at least one statistic is greater than +SMM, and at least one statistic is less than –SMM, then the MCSD ranking crosses, and there is no dominance.*

[11] *The 10th target (i.e. the maximum excess return on the market portfolio) corresponds to the unconditional means, for which we did not find a significant ranking.*

[12] *We also perform the Gibbons, Ross, and Shanken (1989) test for the entire period, and the results reject the market portfolio efficiency relative to the sentiment-beta portfolios. Nevertheless, the results (not reported) need to be used cautiously, given the empirical failure of the normality assumption.*

[13] *Considering that the portfolios are constructed in a similar short/ long fashion, ignoring the transaction costs does not affect the dominance results significantly.*

Again, we point out the importance of the *sentiment* effect in proving that the market portfolio is not efficient, thus allowing investors to benefit from marginally changing their holding of stocks, based on information about the level of optimism/ pessimism in the market.

Conclusions

Over the years, financial theorists have been using various tests to gauge the efficiency of a given portfolio. Second (and higher) degree stochastic dominance tests have become more and more popular, due to their appealing characteristics: economically, they entail meaningful assumptions on the utility functions [non-satiation ($U' \geq 0$) and risk aversion ($U'' \leq 0$)]; statistically, they consider the entire distribution, not only a few moments (as opposed to the mean-variance analysis); another important attribute is the nonparametric approach: the utility function does not have to take any particular form (e.g. quadratic), and the distribution is not restricted (e.g. to normal). Due to the limitations in applying traditional SSD in a portfolio context, the marginal conditional stochastic dominance (MCSD) is considered the appropriate approach.[14]

Under MCSD, the market portfolio (or any other core portfolio) is inefficient if there is a subset of that portfolio that stochastically dominates another subset of the portfolio. Thus, investors are able to improve their expected utility by marginally reallocating the assets in their portfolios (i.e. by altering the relative weights of the dominating/ dominated assets).

In recent decades, part of the financial literature has acknowledged the existence of an investor sentiment effect. Moreover, some researchers have quantified the investor sentiment. We use the Baker and Wurgler (2007) sentiment levels index to show that the *insensitive* stocks marginally and conditionally dominate the *positively* and the *negatively sensitive* stocks, thus demonstrating the market portfolio inefficiency with respect to the sentiment-beta sorted portfolios. Furthermore, we create simple sentiment-arbitrage portfolios, by taking long/ short positions on the sentiment-beta portfolios, conditional on the level of sentiment. Again, we find a dominance pattern between portfolios that confirms the inverse U-shaped outline.

References:

Adler, M. and Dumas, B., 1983, International portfolio choice and corporation finance: A synthesis, *The Journal of Finance*, 38, 925-984.

Baker, M. and Wurgler, J., 2006, Investor sentiment and the cross-section of stock returns, *The Journal of Finance*, 61, 1645-1680.

Baker, M. and Wurgler, J., 2007, Investor sentiment in the stock market, *Journal of Economic Perspectives*, 21, 129-151.

Best, R.J., Best, R.W., and Yoder, J.A., 2000, Value stocks and market efficiency, *Journal of Economics and Finance*, 24, 28-35.

Black, F., 1990, Equilibrium exchange rate hedging, *The Journal of Finance*, 45, 899-909.

Breeden, D.T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics*, 7, 265-296.

Brown, G.W. and Cliff, M.T., 2004, Investor sentiment and the near-term stock market, *Journal of Empirical Finance*, 11, 1-27.

Brown, G.W. and Cliff, M.T., 2005, Investor sentiment and asset valuation, *The Journal of Business*, 78, 405-440.

Carhart, M.M., 1997, On persistence in mutual fund performance, *The Journal of Finance*, 52, 57-82.

Chan, L.K.C., Jegadeesh, N. and Lakonishok J., 1996, Momentum strategies, *The Journal of Finance*, 51, 1681-1713.

Chou, P.H. and Zhou, G, 2006, Using bootstrap to test portfolio efficiency, *Annals of Economics and Finance*, 2, 217-249.

Chow, K.V., 2001, Marginal conditional stochastic dominance, statistical inference, and measuring portfolio performance, *Journal of Financial Research*, 24, 289-307.

Chow, K.V., Denning, K. and Huang, Z., 2008, Style fund investment: performance and efficiency, *International Research Journal of Finance and Economics*, 21, 27-47.

DeSantis, G. and Gerard, B., 1998, How big is the premium for currency risk?, *Journal of Financial Economics*, 49, 375-412.

Fama, E.F. and French, K.R., 1992, The cross-section of expected stock returns, *The Journal of Finance*, 47, 427-465.

Fama, E.F. and French, K.R., 1993, Common risk factors in the returns on stocks and bonds, *The Journal of Financial Economics*, 33, 3-56.

Fama, E.F. and French, K.R., 1996, Multifactor explanations of asset pricing anomalies, *The Journal of Finance*, 51, 55-84.

Fama, E.F. and French, K.R., 1998, Value versus growth: The international evidence, *The Journal of Finance*, 53, 1975-1999.

Gibbons, M.R., Ross, S.A. and Shanken J., 1989, A test of the efficiency of a given portfolio, *Econometrica*, 57, 1121-1152.

Glushkov, D., 2006, Sentiment beta, *Working paper*, Available at SSRN: http://ssrn.com/abstract =862444.

Hadar, J. and Russell, W.R., 1969, Rules for ordering uncertain prospects, *The American Economic Review*, 59, 25-34.

Hanoch, G. and Levy, H., 1969, The efficiency analysis of choices involving risk, *The Review of Economic Studies*, 36, 335-346.

Jegadeesh, N. and Titman, S., 1993, Returns to buying winners and selling losers: implications for stock market efficiency, *The Journal of Finance*, 48, 65-91.

Jegadeesh, N. and Titman, S., 2001, Profitability of momentum strategies: an evaluation of alternative explanations, *The Journal of Finance,* 56, 699-720.

Jewitt, I., 1987, Risk aversion and the choice between risky prospects: The preservation of comparative statics results, *The Review of Economic Studies*, 54, 73-85.

Knez, P.J. and Ready, M.J., 1997, On the robustness of size and book-to-market in cross-sectional regressions, *The Journal of Finance*, 52, 1355-1382.

Kuosmanen, T., 2004, Efficient diversification according to stochastic dominance criteria, *Management Science*, 50, 1390-1406.

Lee, C.M., Shleifer, A. and Thaler, R.H., 1991, Investor sentiment and the closed-end fund puzzle, *The Journal of Finance*, 46, 75-109.

Levy, H., 1992, Stochastic dominance and expected utility: survey and analysis, *Management Science*, 38, 555-593.

[14] *Shalit and Yitzhaki (1994) argue that one limitation in applying traditional stochastic dominance in portfolio context comes from the case of portfolio-choice problems, where it entails infinite pairwise comparisons of alternative probability distributions. Also, once the investor is faced with new alternatives, the whole optimization procedure must be repeated and sometimes, parts of the portfolio cannot be altered.*

Levy, H. and Sarnat, M., 1984, Portfolio and investment selection: Theory and practice, *Prentice-Hall International*.
Lintner, J., 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.
Liu, S., 2006, Investor sentiment and stock market liquidity, *Working Paper*, University of Texas at Austin.
Lo, A.W. and MacKinlay, A.C., 1990, Data-snooping biases in tests of financial asset pricing models, *The Review of Financial Studies*, 3, 431-467.
Loughran, T., 1997, Book-to-market across firm size, exchange, and seasonality: Is there an effect?, *Journal of Financial and Quantitative Analysis*, 32, 249-268.
MacKinlay, A.C., 1995, Multifactor models do not explain deviations from the CAPM, *Journal of Financial Economics* 38, 3-28.
MacKinlay, A.C. and Richardson, M.P., 1991, Using generalized method of moments to test mean-variance efficiency, *The Journal of Finance*, 46, 511-527.
Merton, R.C., 1973, An intertemporal capital asset pricing model, *Econometrica*, 41, 867-887.
Mitchell, M., Pulvino, T. and Stafford, E., 2002, Limited arbitrage in equity markets, *The Journal of Finance*, 57, 551-584.
Post, T., 2003, Empirical tests for stochastic dominance efficiency, *The Journal of Finance*, 5, 1905-1931.
Post, T. and Vliet, P., 2004, Market portfolio efficiency and value stocks, *Journal of Economics and Finance*, 28, 300-306.
Post, T. and Versijp, P., 2007, Multivariate tests for stochastic dominance efficiency of a given portfolio, *Journal of Financial and Quantitative Analysis*, 42, 489-516.
Post, T. and Vliet, P., 2006, Downside risk and asset pricing, *Journal of Banking and Finance*, 30, 823-849.
Qiu, L. and Welch, I., 2004, Investor sentiment measures, *NBER Working Paper Series*, 10794, Available at SSRN: http://ssrn.com/abstract=595193.
Richardson, M. and Smith, T., 1993, A test for multivariate normality in stock returns, *The Journal of Business*, 66, 295-321.
Ross, S., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic* 13, 341-360.
Rothschild, M. and Stiglitz, J. E., 1970, Increasing risk: I. A definition, *Journal of Economic Theory*, 2, 225-243.
Rouwenhorst, K.G., 1998, International momentum strategies, *The Journal of Finance*, 53, 267-284.
Shalit, H. and Yitzhaki, S., 1994, Marginal conditional stochastic dominance, *Management Science*, 40, 670-684.
Shanken, J.A., 1996, Statistical methods in tests of portfolio efficiency: A synthesis, *Handbook of Statistics*, 14, 693-711.
Sharpe, W.F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.
Solnik, B., 1974, An equilibrium model of the international capital market, *Journal of Economic Theory*, 8, 500-524.
Stulz, R., 1981, A model of international asset pricing, *Journal of Financial Economics*, 9, 383-406.
Zhou, G., 1993, Asset-pricing tests under alternative distributions, *The Journal of Finance*, 48, 1927-1942.

Table 1: Distribution properties of the factors' returns

This table reports the distribution parameters and the correlation coefficients for the monthly percentage returns on four stock-market factor portfolios (*RM*, *SMB*, *HML*, and *Mom*) and for the monthly Baker and Wurgler (2007) sentiment levels index, for a time period ranging from January 1966 to December 2010. *RM*, *SMB* and *HML* are the Fama-French factors: *RM* is the market risk premium, *SMB* is Small Minus Big size, while *HML* is High Minus Low book-to-market ratio. *Mom* is the momentum factor, while *Sent* is the measure of investor sentiment. All four stock-market factors follow the description and are obtained from Kenneth French's data library (at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/), while the index of sentiment levels is obtained from Jeffrey Wurgler's website (http://pages.stern.nyu.edu/~jwurgler/).

	Distribution Parameters						Correlation Coefficients				
	RM	*SMB*	*HML*	*Mom*	*Sent*		*RM*	*SMB*	*HML*	*Mom*	*Sent*
						RM	1	0.31	-0.31	-0.13	-0.06
Mean	0.417	0.259	0.381	0.697	0.018	*SMB*		1	-0.24	-0.01	-0.09
Std. Dev.	4.637	3.232	3.008	4.451	0.993	*HML*			1	-0.17	0.05
Skewness	-0.534	0.529	-0.012	-1.404	-0.062	*Mom*				1	0.02
Kurtosis	1.827	5.351	2.316	10.391	0.283	*Sent*					1

Table 2: Summary statistics for sentiment-beta sorted portfolios

The table presents summary statistics for the excess returns (over the one-month Treasury bill rate) on three sentiment-beta sorted portfolios, and for pairwise differences between them.[15] The three sentiment-beta sorted portfolios are obtained by regressing the monthly percentage excess returns of all common stocks (listed on the NYSE, AMEX, and NASDAQ), on the five factors mentioned below and then sorting them according to their sentiment betas. The top 25 percent represent the positively sensitive stocks (β_+^s), the bottom 25 percent indicate the negatively sensitive stocks (β_-^s), while the rest are the insensitive stocks (β_0^s). Monthly data, ranging from January 1966 to December 2010, are obtained from the following sources: stock returns – from the CRSP database; the one-month Treasury bill rate – from Ibbotson Associates; market risk premium, size, value and momentum factors – from Kenneth French's data library (at

[15] *One, two or three asterisks designate significance levels of 10%, 5% and 1%, respectively.*

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/), and the index of sentiment levels – from Jeffrey Wurgler's website (http://pages.stern.nyu.edu/~jwurgler/).

Portfolio	Mean	*t*-statistic	Standard deviation	Skewness	Kurtosis
β_+^s	0.767**	2.49	6.908	0.226	3.671
β_0^s	0.725***	3.26	5.000	-0.221	4.469
β_-^s	0.726**	2.48	6.563	-0.188	2.199
β_+^s - β_0^s	0.042	0.36	2.607	1.302	8.007
β_+^s - β_-^s	0.041	0.50	1.852	0.378	4.985
β_0^s - β_-^s	-0.001	-0.01	2.348	-0.869	4.441

Table 3: MCSD test for sentiment-beta sorted portfolios

The table presents the MCSD test results for pairwise comparisons between three portfolios formed by sorting the sentiment betas, based on the Baker and Wurgler (2007) sentiment levels index. We regress the excess returns of all common stocks (listed on the NYSE, AMEX, and NASDAQ), on the factors mentioned below and then sort stocks according to their sentiment betas. The top 25 percent represent the positively sensitive stocks (β_+^s), the bottom 25 percent indicate the negatively sensitive stocks (β_-^s), while the rest are the insensitive stocks (β_0^s). Monthly data, ranging from January 1966 to December 2010, are obtained from the following sources: stock returns – from the CRSP database; the one-month Treasury bill rate – from Ibbotson Associates; market risk premium, size, value and momentum factors – from Kenneth French's data library (at http://mba.tuck.dartmouth.edu/pages/faculty/ken. french/), and the index of sentiment levels – from Jeffrey Wurgler's website (http://pages.stern.nyu.edu/~jwurgler/). The MCSD ordinates, corresponding to empirical deciles of the market return distribution, are statistically different from zero if their accompanying Z-scores (displayed in parentheses) are greater, in absolute value, than the SMM critical values of 2.560, 2.800 or 3.289, for significance levels of 10, 5 or 1 percent (designated by one, two or three asterisks, respectively). [16]

	ρ									
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
τ_ρ^m	-4.93	-2.54	-1.13	-0.04	1.15	2.05	3.20	4.54	6.07	16.56
β_+^s - β_0^s	-0.252	-0.350	-0.427	-0.462	-0.454	-0.460	-0.305	-0.207	-0.141	0.042
	(-5.16)***	(-6.42)***	(-7.24)***	(-7.11)***	(-6.62)***	(-6.17)***	(-3.42)***	(-2.07)	(-1.34)	(0.36)
β_+^s - β_-^s	0.008	-0.020	-0.005	-0.011	-0.005	-0.015	0.036	0.003	0.006	0.041
	(0.33)	(-0.56)	(-0.11)	(-0.24)	(-0.11)	(-0.29)	(0.60)	(0.05)	(0.08)	(0.50)
β_0^s - β_-^s	0.261	0.330	0.423	0.451	0.449	0.445	0.341	0.210	0.146	-0.001
	(5.79)***	(6.16)***	(7.23)***	(7.32)***	(6.89)***	(6.21)***	(4.24)***	(2.29)	(1.53)	(-0.01)

Table 4: Summary statistics for sentiment-beta sorted portfolios conditional on the level of sentiment

The table presents summary statistics for the excess returns on the portfolios constructed with positively sensitive stocks (β_+^s), insensitive stocks (β_0^s), and negatively sensitive stocks (β_-^s), all with respect to investor sentiment and conditional on the level of sentiment: *High / Medium / Low* .[17] To obtain these three portfolios, we regress the excess returns of all common stocks (listed on the NYSE, AMEX, and NASDAQ), on the factors mentioned below and then sort the stocks according to their sentiment betas. The top 25 percent represent the positively sensitive stocks (β_+^s), the bottom 25 percent indicate the negatively sensitive stocks (β_-^s), while the rest are the insensitive stocks (β_0^s). Monthly data, ranging from January 1966 to December 2010, are obtained from the following sources: stock returns – from the CRSP database; the one-month Treasury bill rate – from Ibbotson Associates; market risk premium, size, value and momentum factors – from Kenneth French's data library (at http://mba.tuck.dartmouth.edu /pages/faculty/ken.french/), and the index of sentiment levels – from Jeffrey Wurgler's website (http://pages.stern. nyu.edu/~jwurgler/).

Portfolio	Sentiment Level	Mean	*t*-statistic	Standard deviation	Skewness	Kurtosis
β_+^s	**High**	-0.225	-0.38	6.831	-0.617	1.898
	Medium	1.075***	2.69	6.776	0.193	2.823
	Low	1.267	1.57	7.390	1.374	7.682

[16] *All the results in this table are reported as percentages.*

[17] *One, two or three asterisks designate significance levels of 10%, 5% and 1%, respectively.*

The sentiment level is High / Medium / Low for values greater than / between / less than the 3rd / 1st quartile of the Baker and Wurgler (2007) levels index.

	High	0.295	0.63	5.345	-0.919	3.349
β_0^s	**Medium**	0.814***	3.05	4.525	-0.606	2.774
	Low	1.096*	1.69	5.930	1.202	6.568
	High	-0.228	-0.38	6.874	-0.520	1.894
β_-^s	**Medium**	1.012***	2.72	6.324	-0.254	1.955
	Low	1.244*	1.68	6.793	0.613	3.225

Table 5: Summary statistics for sentiment-arbitrage portfolios

The table presents summary statistics for the monthly percentage excess returns on three sentiment-arbitrage portfolios and for pairwise differences between them.[18] The three sentiment-arbitrage portfolios are constructed with positively sensitive vs. insensitive, insensitive vs. negatively sensitive, and positively vs. negatively sensitive stocks, based on the Baker and Wurgler (2007) sentiment levels index. We regress the excess returns of all common stocks (listed on the NYSE, AMEX, and NASDAQ), on the factors mentioned below and then sort stocks according to their sentiment betas. The top 25 percent represent the positively sensitive stocks (β_+^s), the bottom 25 percent indicate the negatively sensitive stocks (β_-^s), while the rest are the insensitive stocks (β_0^s). Monthly data, ranging from January 1966 to December 2010, are obtained from the following sources: stock returns – from the CRSP database; the one-month Treasury bill rate – from Ibbotson Associates; market risk premium, size, value and momentum factors – from Kenneth French's data library (at http://mba.tuck.dartmouth.edu/pages/faculty/ken. french/), and the index of sentiment levels – from Jeffrey Wurgler's website (http://pages.stern.nyu.edu/~jwurgler/).

Portfolio	Mean	*t*-statistic	Standard deviation	Skewness	Kurtosis
β_+^s vs. β_0^s	0.165***	2.79	1.326	0.658	7.683
β_0^s vs. β_-^s	0.162***	2.79	1.302	-0.202	11.936
β_+^s vs. β_-^s	-0.003	-0.06	1.125	-1.100	24.207
(β_+^s vs. β_0^s) - (β_0^s vs. β_-^s)	0.003	0.06	1.125	1.100	24.207
(β_+^s vs. β_0^s) - (β_+^s vs. β_-^s)	0.167*	1.80	2.087	1.596	18.630
(β_0^s vs. β_-^s) - (β_+^s vs. β_-^s)	0.165***	2.79	1.326	0.658	7.683

Table 6: MCSD test for sentiment arbitrage portfolios

The table presents the MCSD test results for pairwise comparisons between three sentiment-arbitrage portfolios constructed with positively sensitive vs. insensitive, insensitive vs. negatively sensitive, and positively vs. negatively sensitive stocks, based on the Baker and Wurgler (2007) sentiment levels index. We regress the excess returns of all common stocks (listed on the NYSE, AMEX, and NASDAQ), on the factors mentioned below and then sort stocks according to their sentiment betas. The top 25 percent represent the positively sensitive stocks (β_+^s), the bottom 25 percent indicate the negatively sensitive stocks (β_-^s), while the rest are the insensitive stocks (β_0^s). Monthly data, ranging from January 1966 to December 2010, are obtained from the following sources: stock returns – from the CRSP database; the one-month Treasury bill rate – from Ibbotson Associates; market risk premium, size, value and momentum factors – from Kenneth French's data library (at http://mba.tuck.dartmouth.edu/pages/faculty/ken. french/), and the index of sentiment levels – from Jeffrey Wurgler's website (http://pages.stern.nyu.edu/~jwurgler/). The MCSD ordinates, corresponding to the empirical deciles of the market return distribution, are statistically different from zero if their accompanying Z-scores (displayed in parentheses) are greater, in absolute value, than the SMM critical values of 2.560, 2.800 or 3.289, for significance levels of 10, 5 or 1 percent (designated by one, two or three asterisks, respectively).[19]

	ρ									
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
τ_ρ^m	-4.93	-2.54	-1.13	-0.04	1.15	2.05	3.20	4.54	6.07	16.56
(β_+^s vs. β_0^s) - (β_0^s vs. β_-^s)	-0.006	-0.018	-0.020	-0.033	-0.037	-0.031	-0.046	-0.033	-0.024	0.003
	(-0.35)	(-0.87)	(-0.83)	(-1.18)	(-1.25)	(-0.98)	(-1.37)	(-0.93)	(-0.61)	(0.06)
(β_+^s vs. β_0^s) - (β_+^s vs. β_-^s)	0.102	0.090	0.105	0.102	0.098	0.109	0.091	0.110	0.127	0.167
	(2.74)*	(2.09)	(2.17)	(1.90)	(1.71)	(1.82)	(1.35)	(1.57)	(1.62)	(1.80)
(β_0^s vs. β_-^s) - (β_+^s vs. β_-^s)	0.108	0.108	0.125	0.134	0.135	0.140	0.138	0.144	0.151	0.165
	(3.49)***	(3.25)**	(3.45)***	(3.52)***	(3.35)***	(3.35)***	(2.99)**	(2.99)**	(2.89)**	(2.79)*

[18] *One, two or three asterisks designate significance levels of 10%, 5% and 1%, respectively.*
[19] *All the results in this table are reported as percentages.*